Objective Determining the Part and the Whole in Percent Problems

Warm-Up

Complete each equivalent fraction.

1. $\frac{6}{8} = \frac{\boxed{}}{12}$

2. $\frac{\boxed{}}{16} = \frac{15}{40}$

3. $\frac{6}{15} = \frac{\boxed{}}{10}$

The Big Picture

When you study problems in terms of ratios, like percents, it is important to think about the whole as well as the parts. The whole is not always 100 or 1. And if the whole changes, this changes the percent!

1. Consider the picture of triangles.
a. If the picture shown is 100% of the triangles, draw 50% of the triangles.

b. If the picture shown is 30% of the triangles, draw 100% of the triangles.

2. The given rectangle represents 25% of the whole figure.

a. Draw a rectangle that represents 50% of the whole figure.

b. Draw a rectangle that represents 75% of the whole figure.

c. Draw a rectangle that represents 100% of the whole figure.

3. The figure shown represents 75% of the whole figure.

a. Draw 25% of the figure.

b. Draw 100% of the figure.

Mr. Goodwin, the sixth grade math teacher, asked the class to determine 25% of 44. Five different student responses are shown.

Kendra 👍

Since 25% of 44 means multiplying $\frac{25}{100}$ times the quantity, I used the fraction method.

$\frac{25}{100} = \frac{1}{4}$.

Then, I multiply $\frac{1}{4} \cdot 44 = 11$.

Hank 👍

I like decimals much better than fractions.

$\frac{25}{100} = 0.25$

$0.25 \cdot 44 = 11$

Ryan 👍

25% is easy to do in my head. 50% of 44 is 22.

25% is $\frac{1}{2}$ of 50%, so 25% of 44 is $\frac{1}{2}$ of 22, which is equal to 11.

Simon 👍

Since 25% is the same as $\frac{1}{4}$, I just divided by four.

$44 \div 4 = 11$

PAMELA 👍

I PREFER TO USE THE BENCHMARKS OF 10% AND 5%.

10% OF 44 = 4.4.

20% IS 2 • 10% = 2 • 4.4 = 8.8.

5% IS HALF OF 10% = 2.2.

THEREFORE, 20% + 5% = 8.8 + 2.2 = 11.0

1. Discuss each student method used.

a. When is Kendra's method most efficient to use?

b. When is Hank's method most efficient to use?

c. When is Ryan's method most efficient to use?

d. When is Simon's method most efficient to use?

e. When is Pamela's method most efficient to use?

Ellen said, "All the methods are correct, and everyone got the correct answer, but what if Mr. Goodwin gave us the problem 32% of 732?"

• Kendra said, "My fraction method is not as easy this time."

$$\frac{\overset{8}{\cancel{32}}}{\underset{25}{\cancel{100}}} \cdot \frac{732}{1} = \frac{5856}{25} = 234.24$$

• Hank said,
"32% = 0.32
0.32 · 732 = 234.24
My method is not any more difficult this time."

• Ryan said, "I can still estimate . . . , but my answer will be close, not exact. 32% is close to $\frac{1}{3}$ and $\frac{1}{3}$ of 732 is 244."

• Simon said, "I don't have an easy fraction to use for 32%, so my method works only for certain percents."

• Pamela said, "I can still use my method."

32% = 10% + 10% + 10% + 1% + 1%
10% of 732 = 73.2   1% of 732 5 7.32
73.2(3) = 219.6   7.32(2) = 14.64
                219.6 + 14.64 = 234.24

2. Which method do you prefer with this particular percent of a quantity problem? Explain your thinking.

3. Determine the percent of each quantity.

a. 7% of 80

b. 15% of 55

c. 12% of 320

d. 8% of 300

e. 75% of 240
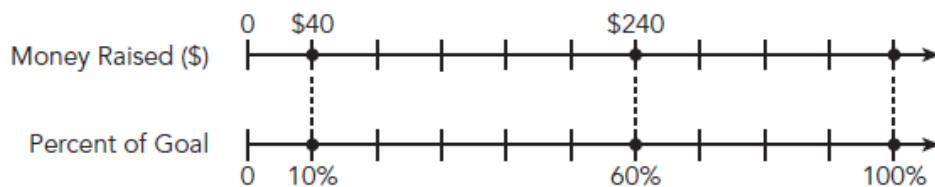
f. 37% of 120

g. 150% of 27

h. 12.5% of 64

Karla is in charge of designing a way to keep a running total of the money raised by her homeroom for the Food Bank project. As of today, her homeroom has raised $240, which is 60% of their goal.

WORKED EXAMPLE

Karla decided to use a double number line to record the money raised and the percent of the goal raised.

The bottom number line represents the percent of the homeroom goal.
The top number line represents the amount of money raised.
Karla's homeroom has raised $240, which is 60% of the goal.

1. How did Karla determine the value that corresponds to 10%?

2. If $240 is 60% of the homeroom goal, what is 100% of the goal? Explain how you determined the answer.
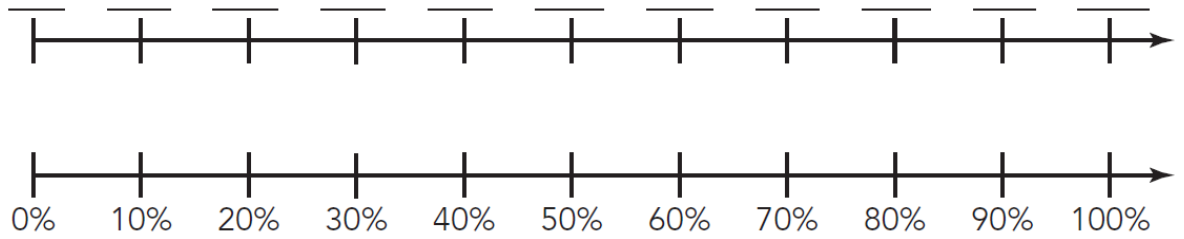
3. Which way of reporting is more informative: the amount of money raised, or the percent of money raised? Explain your thinking.
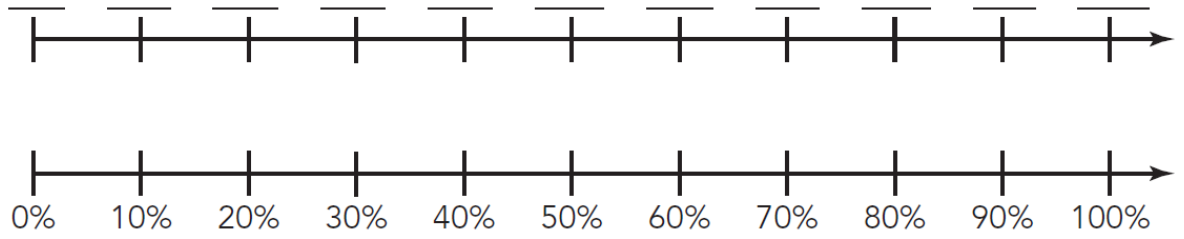
4. Complete each double number line to represent the goals of the other sixth grade homerooms using the information from the table. Write the equivalent dollar amount for each percent shown.

Homeroom	6A	6B	6C	6D	6E	6F
60% of Goal	240	144		288		168
100% of Goal	400		360		120	

a. Homeroom 6B

0%   10%   20%   30%   40%   50%   60%   70%   80%   90%   100%

b. Homeroom 6C

0%   10%   20%   30%   40%   50%   60%   70%   80%   90%   100%

c. Homeroom 6D

0%   10%   20%   30%   40%   50%   60%   70%   80%   90%   100%

d. Homeroom 6E

0%   10%   20%   30%   40%   50%   60%   70%   80%   90%   100%

e. Homeroom 6F

0%   10%   20%   30%   40%   50%   60%   70%   80%   90%   100%
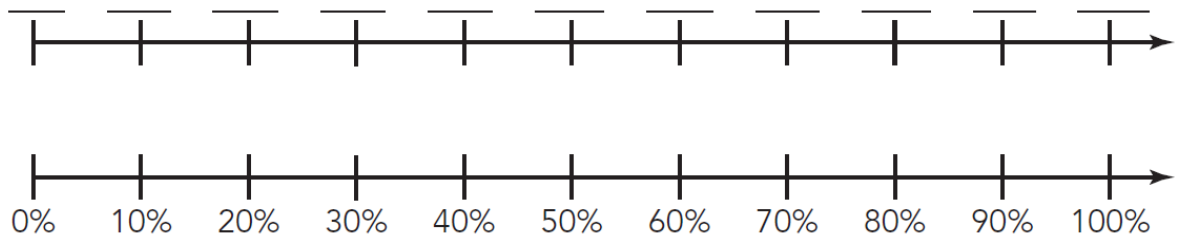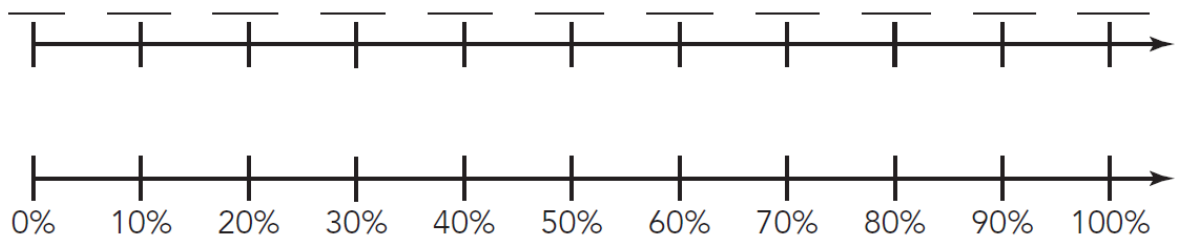
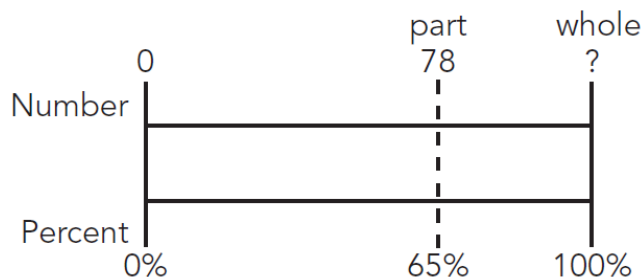# Wholes in Proportions
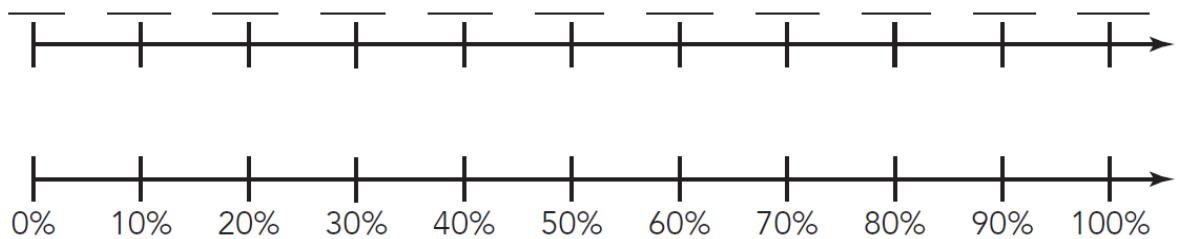
You can use proportions to determine the whole in percent problems.

WORKED EXAMPLE

Carlos is told that 65% of the students, or 78 students, prefer pizza for lunch according to a recent survey. He wants to know how many students were surveyed. He drew the model shown to visualize the problem.

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                                    part      whole
              0                      78          ?
   Number     |_____|
                                     |
                                     |
   Percent    |_____|
              0%                    65%        100%
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He then wrote the proportion and determined that 120 students were surveyed.

$$\frac{part}{whole} \qquad \frac{78}{?} = \frac{65}{100}$$

$$\frac{65}{100} \overset{\div 5}{=} \frac{13}{20} \overset{\times 6}{=} \frac{78}{?}$$

$$\frac{78}{120} = \frac{65}{100}$$

1. How did Carlos determine the total number? Explain Carlos' calculations.

Oscar 👍

Instead of scaling down and then scaling up, it is more efficient to scale up from the start. I can use the structure of this proportion to determine the scale factor.

Since $78 \div 65 = 1.2$, I can scale up in one step by multiplying by 1.2.

$$\overset{\times 1.2}{\underset{\times 1.2}{\frac{78}{?} = \frac{65}{100}}}$$

$$100 \times 1.2 = 120$$

$$\frac{78}{120} = \frac{65}{100}$$

2. Use Oscar's method to determine the missing value.

$$\frac{45}{100} = \frac{126}{?}$$

3. Determine the whole in each situation. Explain your reasoning.

a. The best player on your school basketball team makes 60% of her free throws. If she scored 90 points in a season on free throws, which are worth one point each, how many free throws did she attempt?

b. You got a quiz back and your teacher wrote +16, and 80% at the top. How many points was the quiz worth?

c. Sandy made a 30% deposit on the purchase of a computer. She gave the clerk $168. What is the price of the computer?

d. Your friends ate at a restaurant and left a $2.40 tip. They left a 15% tip. What was the cost of their bill before the tip?

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Review

1. Jai has a 28% free throw rate in basketball. That means when he shoots a free throw he makes a basket 28% of the time. Jai shoots 120 free throws in a season. How many baskets is he likely to make? Use benchmark percents of 1% and 10% to help you determine the answer.

   a. What is 1% of 120?   b. What is 10% of 120?

   c. What is 20% of 120?   d. What is 8% of 120?

2. In Tampa, Florida, the sun shines about 66% of the year. About how many days does the sun shine in Tampa?

3. Bill is painting his room a certain shade of green. The paint is a mixture of 3 parts blue paint to 2 parts yellow paint. To get the correct shade of green, how much yellow paint should he add to 6 quarts of blue paint?

4. LaShaya answered 9 out of 10 questions correctly on her math quiz. Her twin sister LaTeisha answered 22 out of 25 questions correctly on her math test. Did they have the same ratio of correct problems to total problems?

5. Determine each product.

   a. 0.6×95 b. 210×0.75